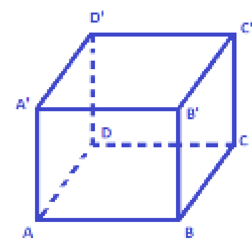
EVALUAREA NAŢIONALĂ PENTRU ABSOLVENŢII CLASEI a VIII-a
Anul școlar 2017 - 2018
Matematică – Rezolvare

Subiectul I

1. $30 - 30:3 = 30 - 10 = 20$
2. $10\ caiete\ \dots\dots 40\ lei$

   $5\ caiete\ \dots\dots\dots\ x\ lei\ \Rightarrow x = \frac{5\cdot 40}{10} = 20\ lei$
3. $A = \{1,2,3,4\}, B = \{1,3,\ x\}\ și\ A \cup B = \{1,2,3,4,5\} \Rightarrow x = 5$
4. $Linia_{mijlocie} = \frac{B+b}{2} = \frac{12+8}{2} = 10\ cm$
5. $V_{ABCDA'B'C'D'} = A_b \cdot H = AB \cdot BC \cdot AA' = 10 \cdot 5 \cdot 4 = 200\ cm^3$
6. $M_a = \frac{1+3+5}{3} = \frac{9}{3} = 3^oC$

Subiectul II



1.
2. $N = 2^{n+3} - 2^{n+2} + 7 \cdot 2^{n+1} - 2^n \ \vdots\ 17$

$$N = 2^{n+3} - 2^{n+2} + 7 \cdot 2^{n+1} - 2^n = 2^n(2^3 - 2^2 + 7 \cdot 2 - 1)$$
$$= 2^n \cdot 17\ \vdots\ 17$$

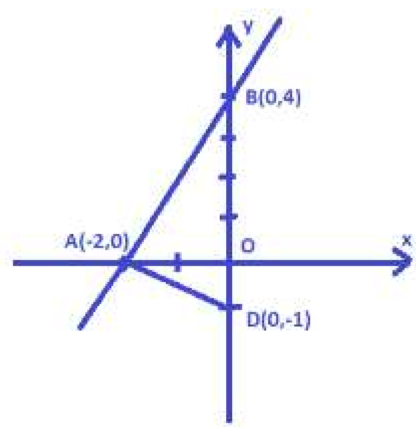
3. $Notăm\ cu\ x\ nr\ de\ elevi\ și\ cu\ y\ suma\ de\ bani$

   $x \cdot 20 + 20 = y\ și\ x \cdot 25 - 5 = y\ \Rightarrow 20x + 20 = 25x - 5\ \Rightarrow 5x = 25 \Rightarrow x = 5\ elevi\quad 5 \cdot 20 + 20 = 120\ lei$
4.

$$a)\ f(x) = 0\ \Rightarrow 2x + 4 = 0\ \Rightarrow x = -2\ \Rightarrow A(-2,0)$$
$$f(0) = 2 \cdot 0 + 4 = 4\ \Rightarrow B(0,4)$$

$b)\ \Delta\ AOB\ dreptunghic\ în\ O\ \Rightarrow$

$$AB^2 = AO^2 + BO^2\ \Rightarrow AB^2 = 2^2 + 4^2 = 20$$
$$\Rightarrow AB = 2\sqrt{5}$$
$$\Delta\ AOD\ dreptunghic\ în\ O\ \Rightarrow$$
$$AD^2 = AO^2 + DO^2\ \Rightarrow AD^2 = 2^2 + 1^2 = 5$$
$$\Rightarrow AD = \sqrt{5}$$
$$BD = BO + OD = 4 + 1 = 5$$
$$În\ \Delta BAD\ verific\ T.Pitagora$$
$$\Rightarrow BD^2 = AB^2 + AD^2\ \Rightarrow 5^2 = \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2$$
$$\Rightarrow 25 = 25\ \Rightarrow DA \perp AB\ \Rightarrow d(D, AB) = \sqrt{5}$$



5. $E(x)=\left(\frac{x+1}{x-3}-\frac{2x^2+3x-3}{x^2-9}+\frac{2x-1}{x+3}\right):\frac{2x^2-18}{x^2+6x+9}$,

Aducem în paranteză la același numitor

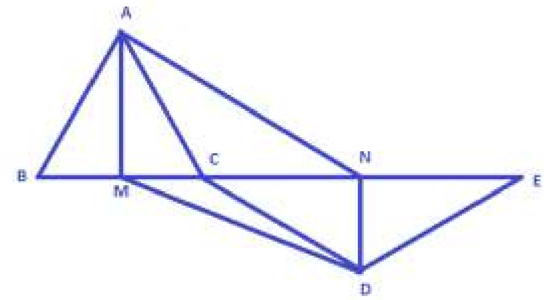
$$E(x)=\frac{(x+1)(x+3)-2x^2-3x+3+(x-3)(2x-1)}{x^2-9}\cdot\frac{x^2+6x+9}{2(x^2-9)}$$

$$E(x)=\frac{x^2+4x+3-2x^2-3x+3+2x^2-7x+3}{(x^2-9)}\cdot\frac{(x+3)^2}{2(x^2-9)}$$

$$E(x)=\frac{x^2-6x+9}{(x^2-9)}\cdot\frac{(x+3)^2}{2(x^2-9)}=\frac{(x-3)^2}{(x^2-9)}\cdot\frac{(x+3)^2}{2(x^2-9)}=\frac{1}{2}$$

Subiectul III

1.



$a)\ m(\sphericalangle DCE)=180^o-m(\sphericalangle BCD)$
$=180^o-150^o=30^o$

$b)\ \Delta ABC\ echilateral\ \Rightarrow AM=\frac{AB\sqrt{3}}{2}$
$=\frac{10\sqrt{3}}{2}=5\sqrt{3}\ cm$

$În\ \Delta DNC\ avem: m(\sphericalangle DNC)=90^o, m(\sphericalangle NCD)=30^o\ \Rightarrow\ m(\sphericalangle NDC)$
$=60^o\ \xRightarrow{T30-60-90}\ ND=\frac{DC}{2}=\frac{10}{2}=5\ cm$

$\xRightarrow{T.Pitagora}\ NC^2=DC^2-ND^2=100-25=75\ \Rightarrow NC=\sqrt{75}=5\sqrt{3}cm$

$În\ \Delta ACM\ și\ CDN\ avem:$
$[AC]\equiv[DC]=10\ cm$
$[AM]\equiv[CN]=5\sqrt{3}\ cm\qquad\xRightarrow{L.L.L.}\ \Delta ACM\equiv\Delta CDN$
$[MC]\equiv[ND]=5\ cm$

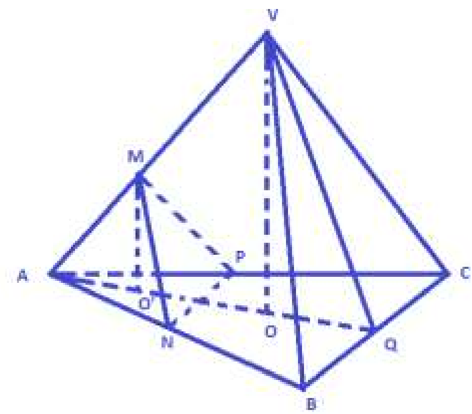
$c)\ A_{AMDN}=A_{AMN}+A_{DNC}=\frac{AM\cdot MN}{2}+\frac{DN\cdot MN}{2}$

$A_{AMDN}=\frac{5\sqrt{3}(5+5\sqrt{3})+5(5+5\sqrt{3})}{2}=\frac{50\sqrt{3}+100}{2}=25\sqrt{3}+50\ cm^2$

$25(\sqrt{3}+2)<95\quad|:5\ \Rightarrow 5\sqrt{3}+10<19\ |-10\ \Rightarrow 5\sqrt{3}<9$

$(5\sqrt{3})^2<9^2\ \Rightarrow 75<81\ (A)$

2.



$a)\ P_{ABC} = 3 \cdot AB = 3 \cdot 12 = 36\ cm$

$b)$ În ΔAVB avem MN linie mijlocie

$\Rightarrow MN \parallel VB$

În ΔACV avem MP linie mijlocie

$\Rightarrow MP \parallel VC$

$VB \cap VC = \{V\}$ și $MN \cap PM = \{M\}$

$\Rightarrow (MNP) \parallel (VBC)$

Dar $VQ \subset (VBC) \Rightarrow VQ \parallel (MNP)$

$$c)\ MN = \frac{VB}{2} = \frac{12}{2} = 6\ cm;\ MA = \frac{AV}{2} = \frac{12}{2} = 6\ cm;\ MP = \frac{CV}{2} = \frac{12}{2} = 6\ cm$$

$$\Rightarrow MA \equiv MN \equiv MP \Rightarrow \Delta ANP \text{ echilateral}$$

$$\Rightarrow MAPN \text{ piramidă triunhiulară regulată}$$

$$MO' = \frac{VO}{2} = \frac{8}{2} = 4\ cm \Rightarrow V_{MANP} = \frac{A_{ANP} \cdot MO'}{3} = \frac{9\sqrt{3} \cdot 4}{3} = 12\sqrt{3}\ cm^3$$

$$V_{VABC} = \frac{A_{ABC} \cdot VO}{3} = \frac{36\sqrt{3} \cdot 8}{3} = 96\sqrt{3}\ cm^3 \Rightarrow V_{VABC} = 8 \cdot V_{MANP}$$

$$V_{MANP} = \frac{p}{100} \cdot V_{VABC} \Rightarrow 8 \cdot p = 100 \Rightarrow p = 12{,}5$$